Exercise 1
1.c
Base case
See point 1
Inductive step

$$A^{n+1} = \begin{pmatrix} \alpha^{n+1} & (n+1)\alpha^n \\ 0 & \alpha^{n+1} \end{pmatrix}$$

$$A^{n+1} = AA^n = \begin{pmatrix} \alpha & 1 \\ 0 & \alpha \end{pmatrix} \begin{pmatrix} \alpha^n & n\alpha^{n-1} \\ 0 & \alpha^n \end{pmatrix}$$

$$= \begin{pmatrix} \alpha\alpha^n & n\alpha\alpha^{n-1} + \alpha^n \\ 0 & \alpha\alpha^n \end{pmatrix}$$

$$= \begin{pmatrix} \alpha^{n+1} & (n+1)\alpha^n \\ 0 & \alpha^{n+1} \end{pmatrix}$$

1.c+1.d

$$f(n) = \sum_{i=1}^{2} (A^n_{i1} + A^n_{i2}) = A^n_{11} + A^n_{12} + A^n_{21} + A^n_{22}$$

$$= \alpha^n + n\alpha^{n-1} + 0 + \alpha^n$$

$$= 2\alpha^n + n\alpha^{n-1}$$

$$2\alpha^n + n\alpha^{n-1} = 2\alpha^n + \frac{1}{\alpha} n\alpha^n = \left(2 + \frac{n}{\alpha}\right)\alpha^n \geqslant An\alpha^n$$

$$\left(2 + \frac{n}{\alpha}\right) \geqslant An$$

$$2\alpha + n \geqslant A\alpha n$$

With $A = \frac{1}{\alpha}$ we have

$$2\alpha + n \geqslant n$$

$$2\alpha \geqslant 0$$

which is true by definition. So

$$2\alpha^n + n\alpha^{n-1} = \Omega(n\alpha^n)\, A = \frac{1}{\alpha}\, a = 1$$

Exercise 2
2.a true solve inequality $(1-A)n^2 - n - 1 \geqslant 0$ for A
2.b true since n^a is above n^b for $a > b$
2.c false
The statement

$$a \geqslant b \text{ is true if } a = b \text{ OR } a > b$$

When we write

$$\text{for all } a \geqslant b$$

we mean

$$\text{for all the combinatios of } a \text{ and } b \text{ such that the statement } a \geqslant b \text{ is true}$$

Some of these combinations make the statement $a \geqslant b$ true for the strict inequality case ($a > b$). But for these combinations $n^a \neq \Theta(n^b)$. In the end, it is not true that

$$n^a = \Theta(n^b) \text{ for all } a \geqslant b$$

(here the quantifier 'for all' plays an important role).
2.d true see lemma 7.5
2.e false since sin oscillates.
Exercise 3
Exercise 4
4.a

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

4.b
Cite theorem 1.7 and note that

$$A^n = A^{n-1} + A^{n-2}$$

4.c

$$w_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, w_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\, w_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, w_3 = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}, w_4 = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix}, w_5 = \begin{pmatrix} 10 \\ 9 \\ 4 \end{pmatrix}$$

4.d

$$B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

4.e

$$B^5_{11} = \left(B^5 w_0\right)_1$$

$$B^5 w_0 = B^4 B w_0 = B^4 w_1 = B^3 w_2 = B^2 w_3 = B w_4 = w_5$$

$$B^5_{11} = (w_5)_1 = 10$$

4.f

$$G = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$