(1)

$$\lim_{x \to -1} \frac{x^3+1}{\sqrt{x^2+2} + \sqrt{3}\,x} =$$

$$= \frac{(x^3+1)}{\sqrt{x^2+2} + \sqrt{3}\,x} \cdot \frac{\sqrt{x^2+2} - \sqrt{3}\,x}{\sqrt{x^2+2} - \sqrt{3}\,x} =$$

$$= \frac{(x^3+1)\left(\sqrt{x^2+2} - \sqrt{3}\,x\right)}{x^2+2-3x^2} = \frac{(x^3+1)\left(\sqrt{x^2+2} - \sqrt{3}\,x\right)}{(-2x^2+2)} =$$

$$= \frac{(x+1)(x^2-x+1)\left(\sqrt{x^2+2} - \sqrt{3}\,x\right)}{2(1-x)(1+x)} = \frac{3\sqrt{3}}{2}$$

(2)

FOR WHICH VALUES OF a, b, c IS

$$A = \begin{bmatrix} a & b & c \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

INVERTIBLE? CALCULATE IT

$$\text{DET} \begin{vmatrix} a & b & c \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} \neq 0 \rightarrow 0 + b - c - a = -a + b - c$$

$$\begin{matrix} -1 & 1 & -1 \\ -c & -c & a-b \\ b-c & a & a \end{matrix}$$

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \frac{1}{\text{DET } A}$$

(3)

$$f(x,y) = x^3 + (x-y)^2$$

$$\partial_1 f(x,y) = 3x^2 + 2(x-y) = 0$$

$$\partial_2 f(x,y) = -2(x-y) = 0$$

$\Rightarrow$ CRITICAL POINT $\begin{cases} x = 0 \\ y = x \end{cases}$

$$\begin{vmatrix} \partial_1^2 f(x,y) = 6x + 2 & \partial_{12}^2 f(x,y) = -2 \\ \partial_{12}^2 f(x,y) = -2 & \partial_2^2 f(x,y) = 2 \end{vmatrix} = 2(6x+2) - 4$$

CRITICAL POINT $(0,0)$ $\Rightarrow$ DET $= 0$

IT MAY BE A SADDLE POINT

$$\Delta f(x,y) = f(x,y) - f(0,0) = x^3 + (x-y)^2 - 0$$

ALONG THE LINE $y = x$

$$\Delta f(x,y) = x^3 > 0 \quad \text{IF } x > 0$$

$$< 0 \quad \text{IF } x < 0$$

SO IT'S A SADDLE POINT

4

$f(x,y) = 4x^2 + 10y^2 \qquad x^2+y^2 \leq 4$

$$\begin{cases} \partial_1 f(x,y) = 8x & x=0 \\ \partial_2 f(x,y) = 20y & y=0 \end{cases}$$

CRITICAL POINT

$\mathcal{L}(x,y,\lambda) = 4x^2 + 10y^2 - \lambda x^2 - \lambda^2 + 4\lambda$

$$\begin{cases} \partial_1 \mathcal{L} = 0 \Rightarrow 8x = 2\lambda x \Rightarrow 2x(4-\lambda) = 0 & x=0 \quad \lambda = 4 \\ \partial_2 \mathcal{L} = 0 \Rightarrow 20y = 2\lambda y \end{cases}$$

$x=0$ $\quad$ CONSTRAIN $y = \pm 2$

$\lambda = 4$ $\quad y=0$ $\quad$ CONSTR $x = \pm 2$

$(0,2) \quad (0,-2) \quad (2,0) \quad (-2,0)$

$f(0,0) = 0$ $\quad$ MIN $\quad$ INT DISK

$f(2,0) = f(-2,0) = 16$

$f(0,2) = f(0,-2) = 40$ $\quad$ MAX $\quad$ BOUND DISK